PMU Placement Considering Data Uncertainty and Redundancy

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***Abstract*— The increasing adoption of Phasor Measurement Units (PMUs, also known as synchrophasors) in the recent acceleration of smart grid technology has posed many interesting questions. One of the main problems that has been explored in many angles has been the PMU placement challenge: On which nodes of a system should the synchrophasors be installed to maximize observability and reliability and reduce costs? Because they are a relatively new addition to the grid, there does not exist much data to establish the reliability values of these components. This is where fuzzy probability theory comes in to introduce a method of dealing with this data uncertainty. This paper aims to build upon an effective PMU placement solution and merge it with fuzzy models for the reliability of the PMUs to explore how the placement decisions are affected. Additionally it will be seen how redundant placements of PMUs on certain critical nodes can improve the system observable reliability compared to isolated placements.**

***Index Terms*— Fuzzy Sets, Phasor Measurement Unit (PMU) Placement, Reliability, Smart Grid, Wide-Area Measurement System (WAMS)**

I. Introduction

With the slow but continuous evolution towards the so called Smart Grid, more and more "smart" components are being added to the existing infrastructure; PMUs (Phasor Measurement Units) being one of the most important ones. PMUs measure voltage, current, and frequency in terms of magnitude and phasor angle at a very high speed, and have gained wide attention within power systems and their application to Wide-Area Measurement Systems (WAMS). Their main purpose is to ensure security and stability of the power system.

Ideally, any network that incorporates PMUs must have full observability, which can be achieved by the strategic placement of PMUs. Statistical uncertainty due to the lack of failure data is a concern when modeling PMUs. With this taken into consideration, paper [1] evaluates alternative methods by incorporating fuzzy evaluation techniques and engineering experience to model PMU reliability. In [1], each component is included in the fuzzy reliability model and then converted into a two-state fuzzy Markov model [1]. This approach will enhance the reliability index and provide lower and upper bounds for PMU reliability.

In [2] the fuzzy reliability model of a phasor measurement unit is divided into eight-states. Similarly, this model was then converted into a two-state Markov representation in series such that the reliability of the entire PMU is easily evaluated. Then the reliability parameters for failure and repair rate were calculated using the fuzzy set triangular membership functions.

In regards to PMU placement, there have been several proposed approaches to evaluate the most effective way of placing the units in a given system including genetic algorithms, Tabu search and integer linear programing [3]-[8]. However, these assume a crisp reliability parameter for the synchrophasors, something that at this point is not accurate since there isn't enough reliability data to support it. In this paper the fuzzy reliability model of PMUs from [1] and [2] will be integrated into the placement method of [8] to evaluate how the results are affected and reach a more accurate placement scheme.

The Reliability-Based Incremental PMU Placement (RBIPP) algorithm described in [8], reaches a predetermined level of system and/or node observability by placing the PMUs in a stepwise fashion on the node which will reduce the system unobservable risk the most. This paper acknowledges that different values for PMU and line reliability (the only two components considered) will in effect result in different PMU placement locations. Although this was briefly tested and proven by using different reliability values for these two components, the use of fuzzy set theory to deal with the inherent uncertainty of PMU reliability values has not been proposed. Furthermore, in some cases it is better to place redundant measurement units on a particular node which is crucial to the system observability; this has not been considered yet.

This paper will describe the fuzzy models in papers [1] and [2], and briefly revisit how they arrived at their results in

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Section II. In Section III a summary of the procedure described in [8] will be presented, along with the modifications and improvements which have been proposed. Section IV will present the results, and finally Section V will have the conclusion.

II. Fuzzy PMU Reliability Models

Paper [1] aims to establish the difference between a crisp and fuzzy reliability model for a PMU. The modules in a fuzzy model are expressed according to the membership function that require interval calculation as fuzzy grades (cut-sets). The PMU is a complex piece of equipment that requires different fuzzy interval calculation methods applied to the different modules. A fuzzy model was developed for all the modules taking data uncertainty into consideration. M0-M6 modules were represented in a series, parallel or series-parallel linkage:

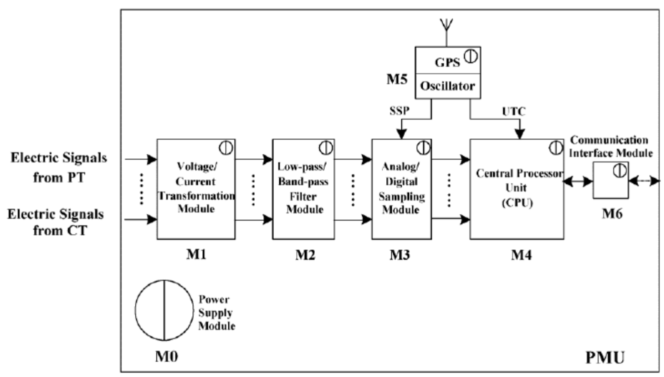

Fig. 1. Structure of PMU [1].

The fuzzy membership function of the unreliability of the PMU was built by the fuzzy interval calculations at different α-cuts ranging from 0.0 to 1.0. The estimated upper and lower bounds of the unreliability for the PMU are calculated using α_0=0.05. The Pseudo-Gauss membership function as seen in Fig. 2 presented in [1] has the mean for unavailability of the PMU at 0.17% with lower bound of 0.052% and upper bound of 0.51%.

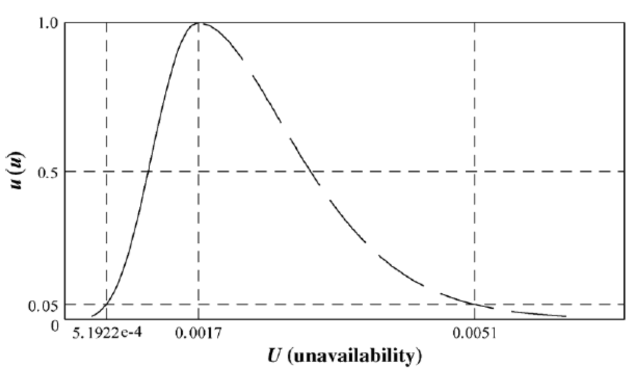

Fig. 2. Membership Function of Unavailability of PMU [1]

Figure 3 shows the eight modules used in [2] to create a series model of the PMU and arrive at their two-state Markov model.

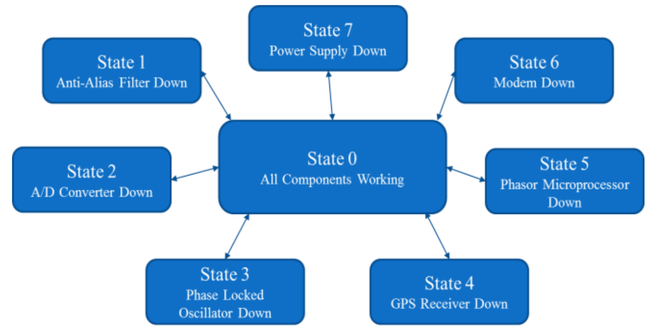

Fig. 3. Eight-State Model From [2].

In the above figure the state 0 represents all the PMU components in working order and states 1-7 are the failing components of the PMU. To calculate availability and unavailability a two-state model is utilized for the whole PMU network.

Figure 4 shows the equivalent two-state model:

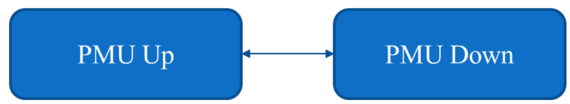

Fig. 4. Two-State Model From [2].

This two-state model is used to calculate the reliability of the whole PMU; the availability and unavailability for their PMU model produced the following fuzzy membership functions:

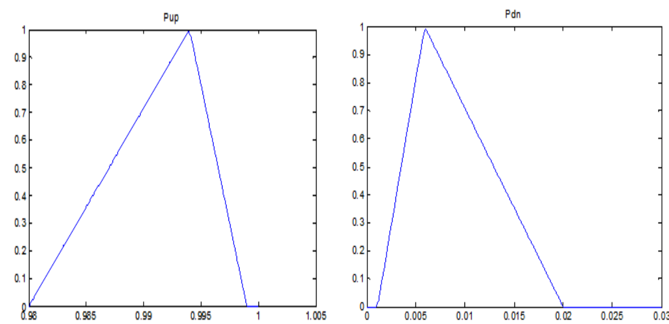

Fig. 5. Availiability and Unavailability From [2]. Pup = 0.994, Pdn = 0.006

III. Reliability-Based Incremental PMU Placement Summary and Modifications

The RBIPP algorithm was chosen as a starting point due to its simplicity, effectiveness and flexibility. The system is first described by a set of vectors and matrices which define the nodes, lines, PMU locations and zero injection nodes. Vector x identifies the PMU locations:

$$x_i = \begin{cases} \text{x}, & number\ of\ PMUs\ on\ node\ i \\ 0, & otherwise \end{cases} \qquad i = 1, \ldots, N \quad (1)$$

The transmission lines are described by matrix A where a_{ji} indicates how many lines between nodes i and j:

$$a_{ji} = \begin{cases} 1, & if\ i = j \\ \text{y}, & number\ of\ lines\ between\ i\ and\ j \\ 0, & otherwise \end{cases} \quad i,j = 1, \ldots, N \qquad (2)$$

Important to note that some systems have redundant connections between nodes, and this paper has taken that into consideration whereas [8] does not. Also, redundant PMUs will be identified on vector x, ([8] represents x as a binary vector indicating whether node has PMU or not).

A node is defined as observable if:

$$u_j = \begin{cases} 1, & if\ \sum_{i=1}^{N} a_{ji} x_i \geq 1 \\ 0, & otherwise \end{cases} \tag{3}$$

Finally, vector z identifies which nodes are zero injection nodes:

$$z_i = \begin{cases} 1, & if\ node\ i\ is\ zero\ injection \\ 0, & otherwise \end{cases} \qquad i = 1, \ldots, N \tag{4}$$

The algorithm will update any unobservable nodes that are adjacent to an observable zero-injection node as per Kirchhoff's Current Law (KCL) further demonstrated in [9].

With these definitions in place, failures of the lines and PMUs can be simulated by simply setting a corresponding value in the x vector or A matrix to zero (or rather subtracting one if the lines or PMUs are redundant) and recalculating the observability. Using the state enumeration method in [10] the state probability of first and second order failures of these two components is calculated by:

$$r_s = \Big(\prod_{\substack{l=1 \\ l \notin K}}^{L} p_l \cdot \prod_{k \in K} q_k\Big) \cdot \Big(\prod_{\substack{n=1 \\ n \notin M}}^{P} p_n \cdot \prod_{m \in M} q_m\Big) \tag{5}$$

L and P represent all the lines and PMUs in the system; K and M represent the sets of failed lines and PMUs. l and n represent the lines and PMUs which have not failed; k and m represent the lines and PMUs which have failed.

Now the unobservable probability of node j can be calculated for any given system (or contingency simulation):

$$w_j = \sum_{s \in ES} r_s \cdot \left[\left(1 - u_j\right) | s\right] = \sum_{s \in ES} w_{j,s} \tag{6}$$

Where ES is the set of all first and second order failure states, r_s is the probability of being in state s, $(1-u_j)\,|\,s$ defines if node j is observable under state s following (3), and $w_{j,s}$ is the resulting unobservable probability of node j under state s.

It follows that the system unobservable probability is the summation of all the unobservable probabilities for all the nodes in the current system:

$$w_T = \sum_{j=1}^{N} w_j \tag{7}$$

The system unobservable probability is what will be used to determine the best placement for the incremental addition of a measurement unit by simply selecting the node which decreases w_T the most. This can be done over and over until a desired level of reliability of system observability is obtained. Paper [8] provides a flow diagram for this procedure:

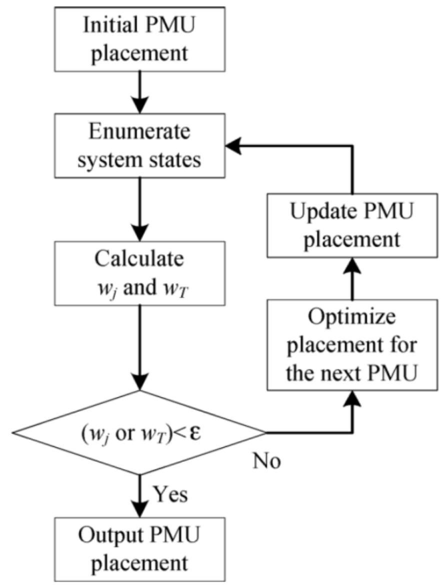

Fig. 6. Flow Diagram of RBIPP [8]

IV. Results

A. Four-Node System

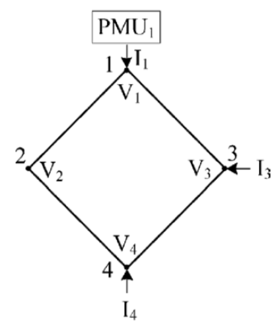

Fig. 7. Four-Node System From [8]

Starting off with the four node system with one PMU and one zero-injection node described in [8] the redundancy addition is evaluated:

Table I
Unobservable Probability of the 4-Node System Considering Placement Redundancy

PMU Nodes	Unobservable Probability				
	Node 1	Node 2	Node 3	Node 4	System
Original (1)	1.670E-03	2.149E-03	2.149E-03	2.628E-03	8.596E-03
(1, 1)	2.784E-06	4.820E-04	4.820E-04	9.610E-04	1.928E-03
(1, 2)	3.583E-06	3.583E-06	2.149E-03	4.836E-04	2.640E-03
(1, 3)	3.583E-06	*2.149E-03*	3.583E-06	5.640E-06	*2.162E-03*
(1, 4)	4.382E-06	*4.611E-06*	4.611E-06	4.382E-06	*1.799E-05*

Interesting to note here is that even though the placement of the additional PMU in node four yielded the best results, a redundant PMU on node one produced better reliability than placing it on node two or three.

Discrepancies were found in the results and are indicated by the italic entries in Table I. For example placing an additional PMU on node three will not change the unobservable probability of node 2 from the original; [8] had a significant decrease in this entry and thus affected the system unobservable probability as well.

Now to briefly evaluate how the fuzzy models of PMU reliability would affect the results for the 4-node system, the lower and upper bounds for PMU reliability of papers [1] and [2] were tested under the same framework:

TABLE II
UNOBSERVABLE PROBABILITY OF THE 4-NODE SYSTEM USING FUZZY PMU RELIABILITY

	System Unobservable Probability				
PMU Nodes	Original (.99833)	Lower Bound from [2] (.98)	Upper Bound from [2] (.9989)	Lower Bound from [1] (.9949)	Upper Bound from [1] (.9995)
Original (1)	8.596E-03	8.188E-02	6.318E-03	2.231E-02	3.919E-03
(1, 1)	1.928E-03	3.478E-03	1.922E-03	2.014E-03	1.920E-03
(1, 2)	2.640E-03	2.218E-02	2.065E-03	6.143E-03	1.461E-03
(1, 3)	2.162E-03	2.172E-02	1.586E-03	5.668E-03	9.820E-04
(1, 4)	1.799E-05	1.673E-03	1.000E-05	1.240E-04	3.000E-06

The results which stand out in this test were that when the PMU reliability was lowered, as in the case of the lower bound from [2] which was 0.98, the system unobservable probability of the redundant PMU placement on node one comes closer to the optimal placement on node 4. On the other hand, when the reliability increases, as in the upper bound from [1] which was 0.9995, the redundant placement is no longer the second best option.

B. 14-Node System

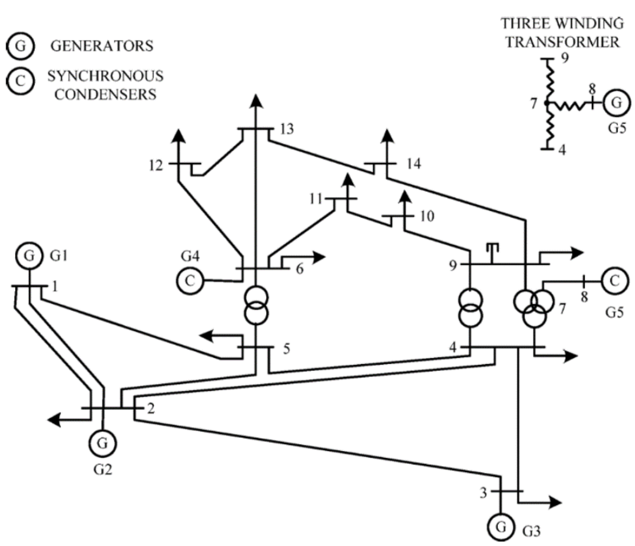

Fig. 8. IEEE 14-Node System [12]

For this system depicted in Figure 8, full observability is obtained with PMUs at nodes 2, 6 and 9 [9]. The same procedure is applied to the IEEE 14-Node system to find the incremental PMU placement which decreases the system unobservable probability the most until a 99.9% observable reliability is reached (which is a 0.1% unobservable probability). Considering PMU placement on redundant and zero-injection nodes, the following results were obtained:

TABLE III
IEEE 14-NODE SYSTEM UNOBSERVABLE PROBABILITY CONSIDERING PLACEMENT REDUNDANCY

Node	PMU Placement				
	2,6,9	2,**4**,6,9	2,4,6,9,**13**	2,4,6,9,**11**,13	2,**2**,4,6,9,11,13
1	*1.670E-03*	1.670E-03	1.670E-03	1.670E-03	2.964E-06
2	1.670E-03	3.542E-06	3.542E-06	3.530E-06	0
3	2.149E-03	4.558E-06	4.558E-06	4.543E-06	1.012E-06
4	*3.548E-06*	0	0	0	0
5	4.566E-06	0	0	0	0
6	1.670E-03	1.670E-03	3.536E-06	0	0
7	2.149E-03	4.558E-06	4.551E-06	4.543E-06	4.536E-06
8	2.628E-03	4.845E-04	4.845E-04	4.845E-04	4.837E-04
9	1.670E-03	2.752E-06	2.747E-06	2.743E-06	2.738E-06
10	2.149E-03	2.149E-03	2.149E-03	4.543E-06	4.536E-06
11	2.149E-03	2.149E-03	2.149E-03	3.530E-06	3.524E-06
12	2.149E-03	2.149E-03	4.551E-06	4.543E-06	4.536E-06
13	2.149E-03	2.149E-03	3.536E-06	3.530E-06	3.524E-06
14	2.149E-03	2.149E-03	4.551E-06	4.543E-06	4.536E-06
Sytem	*2.436E-02*	*1.458E-02*	*6.484E-03*	*2.191E-03*	*5.156E-04*

First of all, because this system has two lines connecting nodes 1 and 2, the results differ from those shown in [8] which did not take this into account. It was determined that the first incremental PMU is to be placed on node 4 rather than node 13 as suggested by [8]. Furthermore, the idea of redundant PMU placement is confirmed in this system as shown by the last PMU placed in node 2. This is intuitive because as can be seen from Figure 8, node 2 is one of the most important nodes in the system due to the quantity of connections it has. Applying the same upper and lower bounds for PMU reliability from [1] and [2] as in case A:

TABLE IV
UNOBSERVABLE PROBABILITY OF THE 14-NODE SYSTEM USING FUZZY PMU RELIABILITY FROM [2]

Lower Bound from [2] (.98)		Upper Bound from [2] (.9989)	
PMU Placement	System	PMU Placement	System
2,6,9	2.448E-01	2,6,9	1.752E-02
2,6,9,**9**	1.436E-01	2,**4,**6,9	1.059E-02
2,6,**6**,9,9	6.507E-02	2,4,6,9,**13**	4.757E-03
2,**2,**6,6,9,9	8.442E-03	2,4,6,9,**11**,13	1.601E-03
2,2,**4,**6,6,9,9	5.234E-03	2,**2**,4,6,9,11,13	4.980E-04
2,2,4,6,6,9,9,**13**	2.452E-03		
2,2,4,6,6,9,9,**10,**13	8.590E-04		

As was determined on the 4-node system, when the PMU reliability is lowered, the redundant placement becomes more crucial; this makes sense because with more unreliable PMUs, the most critical nodes need a backup. As can be seen from the lower bound reliability number from [2], the first three additional PMUs are placed on redundant nodes. Also, as was expected, it takes more PMUs to reach the same system observability (two additional ones compared to original).

For the upper bound reliability the placement order stays the same as the original, but as can be seen the system unobservable probability at each stage is decreased.

TABLE V
UNOBSERVABLE PROBABILITY OF THE 14-NODE SYSTEM USING FUZZY PMU RELIABILITY FROM [1]

Lower Bound from [1] (.9949)		Upper Bound from [1] (.9995)	
PMU Placement	System	PMU Placement	System
2,6,9	6.555E-02	2,6,9	1.032E-02
2,**4**,6,9	3.870E-02	2,**4**,6,9	6.382E-03
2,4,6,9,**13**	1.699E-02	2,4,6,9,**13**	2.946E-03
2,4,6,9,**11**,13	5.866E-03	2,4,6,9,**11**,13	9.880E-04
2,**2**,4,6,9,11,13	7.350E-04	*2,* ***2****,4,6,9,11,13*	*4.870E-04*

Because the lower and upper bound reliability figures for [1] are closer to the original (0.99833), the order was not altered but of course the unobservable probability was affected. For the upper bound of 0.9995 only three additional PMUs are needed to reach the required 99.9% observable reliability. The last incremental placement is shown in table V to prove that even with an extremely reliable PMU, redundant placement still gives a better system observable reliability than limiting the placement of the synchrophasors to nodes without a PMU.

C. RTS 96 Test System

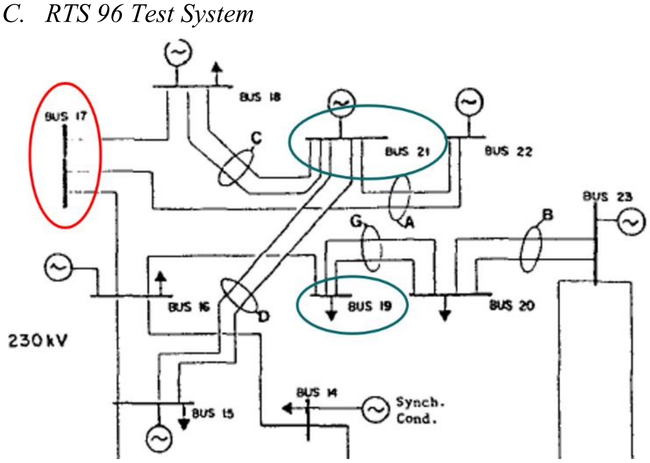

Fig. 9. IEEE One Area RTS-96 [13]

Paper [8] states that for full initial observability for the RTS 96 Test System [13], PMUs should be placed on nodes 101, 102, 108, 113, 119, 121, 201, 202, 208, 216, 221, 223, 302, 305, 308, 315, 316 and 323. However, looking at Area One of this system it can be seen that these will not provide the full observability which is desired. As an example, it can be seen in Figure 9 the PMUs on bus 19 and 21 do not provide observability of bus 17 because there isn't a direct connection or an indirect zero-injection connection.

It was determined that for full observability 5 additional PMUs are needed at 209, 309, 116, 215 and 124 (in order of increasing importance).

To reach the goal of less than 0.1% of system unobservable probability 38 PMUs are needed, they are listed here in order of placement:

101, 102, 108, 113, 119, 121, 201, 202, 208, 216, 221, 223, 302, 305, 308, 315, 316, 323, **209, 309, 116, 215, 124, 110, 210, 322, 310, 203, 123, 220, 319, 122, 303, 109, 207, 307, *302*, 103.**

The resulting system unobservable probability with all the 38 PMUs is 0.0118%. Also, again it is seen in this system that there is a redundant placement of PMU at node 302.

V. CONCLUSION

When initially placing synchrophasors on a grid, maximum observability, and thus system observable reliability, is desired. Due to the data uncertainty regarding the reliability of PMUs, the placement order can be vastly affected depending on which numbers are used. Keeping this in mind, as more data is collected, the RBIPP approach will provide an increasingly clear and cost effective way of incrementally adding the measurement units on the grid. However, it has been proven that even if a node already has a PMU installed on it, it shouldn't be discarded from the potential pool of candidates for the next PMU addition. Even though initially it might seem counter-intuitive, when analyzed more in depth it makes sense that crucial nodes in a system could require a backup unit.

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